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Strip foundations on a cross-anisotropic soil layer subjected to dynamic loading

G. GAZETAS*

A rigorous semi-analytical formulation is presented to study the static and dynamic response of rigid strip footings supported on the surface of a horizontally layered soil deposit. Each layer is modelled as a homogeneous cross-anisotropic medium with a vertical axis of material symmetry while the loading consists of harmonically time-varying horizontal or vertical forces and moments. The solution, based on an experimentally verified relationship among the anisotropic soil parameters that uncouples the wave equations in closed-form, is exact in that it properly accounts for the true boundary conditions at the layer interfaces and the surface. Comprehensive parametric studies are presented in the form of normalized displacement–load or rotation–moment ratios as functions of dimensionless geometric and material parameters. Simple and sufficiently accurate formulas of direct practical applicability are also given for static displacements and resonant frequency factors. The results clearly demonstrate the significance of soil anisotropy in determining undrained static and dynamic response of foundations; soils with a large ratio of horizontal to vertical Young's moduli experience smaller static displacements and quite different dynamic response characteristics from equivalent isotropic soil deposits.

L'article présente une formulation semi-analytique rigoureuse pour étudier la réponse dynamique et statique de semelles filantes rigide reposant à la surface de sédiments de sols à couches horizontales. Chaque couche est modélisée sous la forme d'un milieu homogène de section anisotrope ayant un axe vertical de symétrie matérielle, tandis que la charge est constituée de forces et moments verticaux ou horizontaux à variation harmonique dans le temps. La solution, basée sur une relation vérifiée expérimentalement entre les paramètres de sols anisotropes qui découple les équations d'onde de forme fermée, est exacte en ce qu'elle tient bien compte des conditions de limite vraie aux interfaces des couches et à la surface. L'article présente des études paramétriques complètes sous la forme de rapports rotation-moment ou déplacement-charge normalisés en tant que fonctions de paramètres matériels et géométriques adimensionnels. L'article présente également des formules simples et suffisamment précises ayant une application pratique directe pour des facteurs de fréquence de résonance et des déplacements statiques. Les résultats montrent bien l'importance que représente l'anisotropie des sols pour la détermination de la réponse dynamique et statique non drainée de fondations; dans le cas de sols dont le rapport des modules d'élasticité de Young horizontal/vertical est important, les déplacements statiques sont moindres et les caractéristiques de résponse dynamique sont totalement différents de celles de sédiments de sols isotropiques équivalents.

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NOTATION

A_V	dimensionless frequency factor $(=\omega B\sqrt{(\rho/E_V)})$
B	half of the foundation width
D_{ij}	elastic stiffness parameters of an anisotropic material (equation (5))
E_V, E_H	Young's modulus in vertical and horizontal directions
G_{VH}, G_{HH}	shear modulus in vertical and horizontal planes
h	ω/α
H	thickness of soil stratum
k	ω/β
L	pseudo-distortional wave potential
M_o	moment on a rigid foundation (with respect to axis 0y)
n	E_H/E_V
N	pseudo-dilatational wave potential
P_H	horizontal force of a rigid foundation
P_V	vertical force of a rigid foundation
r	dE_V/dz
u	horizontal displacement
w	vertical displacement
x	horizontal co-ordinate
z	vertical co-ordinate
α	$\sqrt{(D_{33}/\rho)}$
β	$\sqrt{(G_{VH}/\rho)}$
δ_H	horizontal displacement of a rigid foundation
δ_V	vertical displacement of a rigid foundation
$\varepsilon_x, \varepsilon_z, \gamma_{xz}$	normal and shear strain components
ν_{VH}	Poisson's ratio for effect of vertical strain on horizontal strain
ν_{HH}	Poisson's ratio for effect of horizontal strain on complementary horizontal strain
ξ	soil damping ratio
ϕ_o	angle of rotation of a rigid foundation (with respect to axis 0y)
ρ	soil density
$\sigma_x, \sigma_z, \tau_{xz}$	normal and shear stress components
ω	frequency of vibration (rad/s)

INTRODUCTION

A large body of experimental evidence suggests that soils and rocks in nature invariably exhibit some degree of anisotropy in their response to stresses. Natural clay deposits, for instance, during their formation by sedimentation and subsequent one-dimensional consolidation acquire a fabric that is characterized by particles or particle-units oriented in a horizontal arrangement. This preferred orientation and the resulting electrochemical bonds among the clay particles are the cause of cross-anisotropic deformational behaviour, as has been demonstrated in numerous experimental investigations (Ward, Samuels & Gutler, 1959, 1965; Barden, 1971, 1972; Franklin & Mattson, 1972; Kirkpatric & Rennie, 1972; Saada & Ou, 1973; Gibson, 1974; Atkinson, 1975; Saada, Bianchini & Shook, 1978; Yong & Silvestri, 1979). This anisotropy becomes marked in heavily over-consolidated clays, such as the London clay; an extreme case is the eventual formation of strongly anisotropic laminated shales, slates and mudstones.

Sands also exhibit deformational anisotropy, which arises chiefly from the influence of gravity and particle shape on the deposition process. Experimental investigations have revealed that sand particles have a strong tendency to adopt preferential orientation with the maximum dimension aligned in a horizontal plane (Willoughby, 1967, Parkin, Gerrard & Willoughby, 1968); thus the number of contact points per unit horizontal area is smaller than that per unit vertical area (Rowe, 1962; Barden, 1963). Such a fabric is conducive to greater radial than vertical compressibilities, as was shown by Gerrard (1968) and Arthur & Menzies (1972). A set of equal-diameter spheres in a hexagonal array have cross-anisotropic properties (Gassman, 1953).

There is a growing awareness of the need to account for the influence of soil cross-anisotropy when estimating foundation settlements or distribution of stresses in the ground. This is well reflected in a recent state-of-the-art presentation on foundation behaviour by Burland, Broms & de Mello (1977) as well as a number of publications dealing with the mathematical modelling of soil anisotropy and its implications on foundation response. However, essentially all of these studies are limited to considering only static loading conditions. The interest in designing foundations subjected to dynamic loads (such as those arising from supported machinery, sea waves, earthquakes and ground-transmitted traffic or blast vibrations) makes necessary the study of the dynamic interaction of foundations with cross-anisotropic soil. Accordingly, the objective of this Paper is to assess the effect of soil anisotropy on the response of strip foundations to dynamic vertical, horizontal and rotational excitation.

In order to reach conclusions of the widest possible applicability with actually encountered soil deposits, this Paper idealizes soil as a layered medium, with each layer being a linearly elastic cross-anisotropic continuum having a vertical axis of symmetry and exhibiting linear hysteretic-type of damping when subjected to dynamic stresses. A semi-analytical solution is presented based on a transformation that uncouples the Navier-type governing equations in terms of pseudo-distortional and pseudo-dilatational wave potentials. Analytical expressions can, thereby, be obtained for displacements and stresses in each layer and the correct boundary conditions at layer interfaces can be enforced in continuous form. On the other hand, in order to satisfy the mixed boundary conditions at the surface, a numerical scheme has been devised involving discretization into a number of uniformly spaced nodal points and use of a so-called fast Fourier transform algorithm to perform the pertinent integrations. As a result, the method possesses the flexibility of the numerical techniques (like the finite element method) in properly handling any prescribed mechanical behaviour of the soil–foundation interface; thus the two commonly assumed extreme cases of adhesive and frictionless contact can be studied almost as easily as the more realistic case of a contact obeying Coulomb's friction law.

Yet, owing to its analytical character, the presented method is devoid of a crucial limitation of the finite element techniques, namely, their inability accurately to model the radiation of wave energy at very large horizontal and vertical distances from the oscillating foundation. It is true, of course, that special energy-absorbing boundaries, such as those developed recently by Valliapan, White & Lee (1977) for a cross-anisotropic material, can provide partial remedy and lead to a more or less acceptable dynamic finite element formulation. Nonetheless, besides their greater accuracy, analytical solutions offer a clear economic advantage in terms of both computer storage and time. This makes quite feasible the performance of comprehensive parametric studies aimed at evaluating the relative importance of anisotropy with a variety of characteristic soil profiles ranging from homogeneous halfspace to shallow stratum underlain by rigid rock. The results of such a study are presented here in the form of graphs and simple formulas of direct practical applicability, and are compared with other relevant solutions.

SUMMARY OF PREVIOUS WORK

Several studies related to the present problem have been published and it is useful to review them briefly before proceeding to our analyses.

Studies related to the static problem

Michell (1900), extending the work of Boussinesq and Cerruti, presented a solution for stresses and displacements in a halfspace exhibiting hexagonal anisotropy[1] and subjected to a vertical or horizontal point force. Much later, his solution was simplified and made popular by Barden (1963) who proposed the cross-anisotropic halfspace as an improved mathematical model for natural soils, clays or sands. The major conclusion of Barden's study is that, for a realistic range of Poisson's ratios, as the ratio n of the horizontal to vertical Young's moduli increases so does the load-spreading capacity of the soil; hence, both stress concentration along the load axis and surface settlement decrease.

Extensive studies on static interaction of foundations with anisotropic soils were presented by Gerrard & Harrison (1970a, b); they reported complete solutions (stresses, strains, displacements) for cross-anisotropic and orthorhombic homogeneous halfspaces carrying circular or strip foundations of infinite or zero rigidity that are loaded by vertical, horizontal and moment forces. Their results, although in a somewhat complicated mathematical form, have contributed much to current understanding of the behaviour of anisotropic soils. The same authors (Harrison & Gerrard, 1972) established the equivalence between earth reinforced by means of thin sheets (or bars) of a stiffer material, and a cross-anisotropic (or orthorhombic) homogeneous medium. As a concept, reinforced earth had been proposed by Casagrande to model naturally stratified soil deposits; Westergaard (1938) worked out the idea and presented solutions for a halfspace so stiff in the horizontal direction, that no lateral strains could occur. Such a medium is, in fact, a cross-anisotropic material with $n = \infty$ and $\nu_{VH} = 0$. Particular types of cross-anisotropy, characterized by three instead of five independent parameters, were studied by Wolf (1935) and Milovic (1972).

Analytical results for an incompressible cross-anisotropic halfspace whose modulus, E_V, varies with depth according to

$$E_V(z) = rz \tag{1}$$

while m, n, ν_{VH} and ν_{HH} remain constant, have been published by Gibson (1974), Gibson & Kalsi (1974) and Gibson & Sills (1974). Such a continuum, which is of direct practical interest when determining undrained settlements of foundations on deep saturated clay deposits, was found to behave as a 'Winkler' medium; regardless of geometry of the loaded area, surface settlement is directly proportional to the applied normal pressure. With very good accuracy their main result simplified to

$$w(x, y) = q(x, y)\frac{4-n}{r[(4-n)\,G_{VH}/E_V+1]} \tag{2}$$

where, in this case, n and G_{VH}/E_V fully describe soil cross-anisotropy, since for an incompressible medium

$$\begin{aligned} \nu_{VH} &= 1/2 \\ \nu_{HH} &= 1-n/2 \end{aligned} \tag{3}$$

As a direct consequence, soil reactions against rigid smooth foundations of any shape are uniform. This is also true with isotropic incompressible media obeying equation (1) (Gibson, 1974). It appears that this interesting conclusion can be generalized: Sveklo (1970) has found that in the case of rigid foundations having circular, elliptical or elliptical–paraboloidal contact with an anisotropic homogeneous halfspace, soil reactions are independent of both type and degree of anisotropy—a conclusion that does not apply to foundation settlements.

Finally, Hooper (1975) studied with a finite element formulation the interaction of circular flexible rafts in adhesive contact with a cross-anisotropic layered stratum. Using material properties appropriate for the overconsolidated London clay (Ward *et al.*, 1959; Ward, Marsland & Samuels, 1965; Uriel & Kañizo, 1971; Gibson, 1974; Atkinson, 1975), he demonstrated that total and differential foundation settlements may be reduced by about 40% under undrained conditions if cross-anisotropy is taken into account; under drained conditions this reduction is only 5–20%, depending on the type of soil profile assumed (homogeneous or linearly heterogeneous).

Studies related to the dynamic problem

Kirkner (1979) presented an analytical study of the steady-state response of a circular foundation on a cross-anisotropic halfspace whose elastic parameters satisfied a certain constraint relationship (equation (6)), originally suggested by Carrier (1946). The motivation in establishing such a relationship was one of convenience: the three equations of motion can uncouple and thereby be solved analytically. Several other researchers have also used equation (6) to restrict material anisotropy. For instance, Payton (1975) obtained solutions for dynamic displacements due to a concentrated force suddenly applied at a point within an infinite elastic space; and Valliapan *et al.* (1976) derived dashpot constants of energy-absorbing boundaries for a plain-strain finite-element formulation.

An interesting conclusion of Kirkner's work is that anisotropy may have different effects at high

[1] The terms *hexagonal anisotropy* as well as *transverse isotropy* are used in the literature and are equivalent to the term *cross-anisotropy* employed in this Paper.

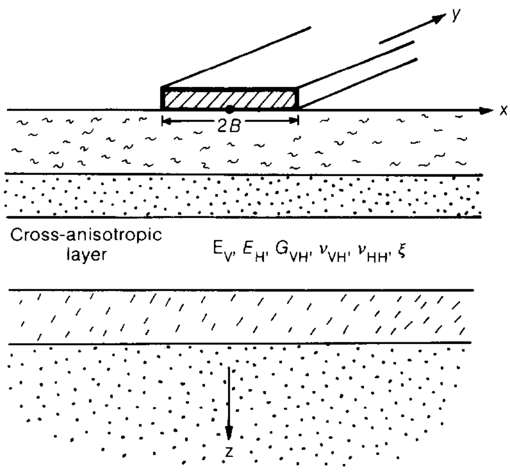


Fig. 1. Soil profile and footing diagram

and low frequencies of vibration. For example, on an incompressible medium a circular footing experiences static horizontal displacements that decrease as the lateral soil stiffness increases relative to the vertical, i.e. as $n = E_H/E_V$ increases; at high frequencies, however, larger dynamic displacements are associated with larger n values. Such a conclusion is of practical significance since the constraint relationship on which the analysis was based has been shown to be satisfied with sufficient accuracy by many soils, as demonstrated in this Paper.

DEFORMATIONAL ELASTIC SOIL ANISOTROPY

Mathematical modelling

The use of an elastic model for soils at low working stresses has received much attention (e.g. Poulos & Davis, 1974). It is generally accepted that for stress changes imposed by shallow foundations linear elastic theory provides an adequate engineering model enabling the prediction of settlements to a sufficient degree of accuracy. This is especially true e.g. for heavily overconsolidated clays (Wroth, 1971) and brittle sensitive clays (Yong & Silvestri, 1979) stressed below their yield limit. Moreover, elastic models lead to satisfactory answers in problems of dynamic soil–foundation interaction; for small dynamic strains such as those developed beneath machine foundations, soils exhibit an approximately elastic behaviour (Richart, Woods & Hall, 1970), while the local soil non-linearities arising during strong earthquake or wave excitation 'do not significantly affect the response of the structure' (Kausel, Roesset & Christian, 1976).

A cross-anisotropic elastic material is characterized by five independent constants: two Young's moduli E_V and E_H, a shear modulus G_{VH}, and two Poisson's ratios ν_{VH} and ν_{HH} (Lekhnitskiĭ, 1963; Pickering, 1970). With the vertical axis z being the axis of elastic symmetry (Fig. 1), the stress–strain relationships appropriate for plane-strain conditions take the form

$$\begin{aligned} \sigma_x &= D_{11}\varepsilon_x + D_{13}\varepsilon_z \\ \sigma_y &= D_{12}\varepsilon_x + D_{13}\varepsilon_z \\ \sigma_z &= D_{13}\varepsilon_x + D_{33}\varepsilon_z \\ \tau_{xz} &= G_{VH}\gamma_{xz} \end{aligned} \tag{4}$$

where the four elastic coefficients D_{11}, D_{12}, D_{13} and D_{33} are related to the Young's moduli and Poisson's ratios

$$\begin{aligned} D_{11} &= (E_H/a)(1-n\nu_{VH}^2) \\ D_{12} &= (E_H/a)(n\nu_{VH}^2+\nu_{HH}) \\ D_{13} &= (E_H/a)\nu_{VH}(1+\nu_{HH}) \\ D_{33} &= (E_V/a)(1-\nu_{HH}^2) \end{aligned} \tag{5}$$

in which

$$\begin{aligned} n &= E_H/E_V \\ a &= (1+\nu_{HH})(1-\nu_{HH}-2n\nu_{VH}^2) \end{aligned} \tag{5a}$$

Thermodynamic considerations require that the strain energy in an elastic material due to all possible stress fields be non-negative. This imposes on the acceptable range of the elastic parameters certain restrictions (Hearmon, 1961; Pickering, 1970; Gibson, 1974; Hooper, 1975).

Constraint relationship

The constraint relationship which has been proposed by Carrier (1946) defines a subset of all cross-anisotropic materials by making the shear modulus G_{VH} a function of the other elastic parameters

$$G_{VH} = \frac{D_{11}D_{33}-D_{13}^2}{D_{11}+2D_{13}+D_{33}} \tag{6}$$

Thus the number of independent elastic parameters reduces to four, and the equations of motion can uncouple and be solved in closed form—a convenient simplification which motivated the adoption of equation (6). For a material with $\nu_{VH} = \nu_{HH} \equiv \nu$ and $E_H = E_V \equiv E$, equation (6) reduces to the well-known relationship between moduli and Poisson's ratio of isotropic media: $2G_{VH} \equiv 2G = E(1+\nu)$; that is, isotropy can be recovered from the proposed constraint relationship.

From a physical standpoint, significant experimental evidence has come to support the use of equation (6) with many types of soil (Gazetas, 1980b). This rather unexpected but most welcome conclusion was reached by testing the validity of equation (6) against numerous published experimental data which are summarized in Table 1.

Using the reported values of n, ν_{VH}, ν_{HH} and E_V, shear moduli G_{VH} were computed from the constraint relationship; they are also depicted in Table 1 for comparison with the experimental values.

The performance of equation (6) appears to be quite satisfactory. In several cases calculated and measured moduli are nearly identical and in no case do they differ by more than 20%. This is believed to be within the range of possible error of the reported G_{VH} values, which are frequently derived from Young's moduli E_V, E_H and $E_{45°}$ (Gibson, 1974). Because, as pointed out by Pagano & Halpin (1968) and Saada & Bianchini (1977), when samples cut with their axis inclined at 45° to the vertical are tested in triaxial compression, extraneous bending and shear end effects are generated; as a result the recorded $E_{45°}$ underestimates the actual modulus by an amount which increases with the degree of anisotropy. Consequently, G_{VH} may in reality be somewhat larger than the reported values indicate, especially for the heavily overconsolidated London clay. This might further improve the agreement with values derived from the constraint relationship.

In conclusion, equation (6) appears to be quite a realistic assumption for a number of clays and its use in the theory that is presented here is thus fully justified.

Table 1. Anisotropic elastic constants of clays and evaluation of the constraint relationship

Description of soil	Reference	Measured values				Computed (equation (6))
		n	ν_{VH}	ν_{HH}	G_{VH}/E_V	G_{VH}/E_V
Heavily overconsolidated London clay (Ashford) (undrained loading)	Ward *et al.* (1959) Ward *et al.* (1965) Gibson (1974)					
Depth: 30 ft		1·35	0·50	0·325	0·35*	0·355
Depth: 50 ft		1·59	0·50	0·205	0·37*	0·41
Average of all samples		1·80	0·50	0·08	0·38*	0·46
Heavily overconsolidated London clay (Barbican Arts Centre) (drained loading)	Atkinson (1975)	2·00	0·19	0·00	0·536†	0·553
Lightly overconsolidated kaolinite clay (Florida Edgar plastic kaolin) (undrained loading)	Saada *et al.* (1978)					
$\sigma_c' = 40$ lb/in^2 water content = 40·7%		1·25	0·50	0·375	0·356	0·364
$\sigma_c' = 60$ lb/in^2 water content = 38·7%		1·355	0·50	0·322	0·362	0·378
Normally consolidated illite clay (Grundite) (undrained loading)	Bianchini (1980)					
$\sigma_c' = 70$ lb/in^2 $w = 29·5\%$		1·17	0·50	0·415	0·355	0·353
$\sigma_c' = 60$ lb/in^2 $w = 38·1\%$		1·13	0·50	0·436	0·322	0·310
Colorado clay shale (drained loading)	Kaarsberg (1968)	1·38	0·197	0·266	0·423	0·524
Sensitive, naturally cemented Champlain sea clay (Canada) (drained loading)	Yong & Silvestri (1979)	0·62	0·35	0·20	0·205*	0·187

* Estimated from $E_{45°}$ using equation (21) of Gibson (1974).

† Estimated by the Author from undrained tests on the basis of the theoretical formulas of Uriel & Kañizo (1971), as explained in Appendix 1.

PROBLEM FORMULATION AND SOLUTION: CROSS-ANISOTROPIC LAYERED HALFSPACE

Within each layer, dynamic equilibrium under plane-strain conditions, as is appropriate for strip loading, requires that the stresses satisfy the equations

$$\frac{\partial \sigma_x}{\partial x}+\frac{\partial \tau_{xz}}{\partial z}=\rho\frac{\partial^2 u}{\partial t^2}$$
$$\frac{\partial \sigma_z}{\partial z}+\frac{\partial \tau_{xz}}{\partial x}=\rho\frac{\partial^2 w}{\partial t^2} \tag{7}$$

where the displacements $u(x,z)$ and $w(x,z)$ are related to the strain components $\varepsilon_x(x,z)$, $\varepsilon_z(x,z)$ and $\gamma_{xz}(x,z)$ as follows

$$\varepsilon_x=\frac{\partial u}{\partial x}$$
$$\varepsilon_z=\frac{\partial w}{\partial z} \tag{8}$$
$$\gamma_{xz}=\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}$$

Equations (4), (7) and (8) describe completely the time and space variation of stresses, strains and displacements. Upon eliminating stresses and strains, the following Navier-type coupled equations of motion are obtained

$$\rho\frac{\partial^2 u}{\partial t^2}=D_{11}\frac{\partial^2 u}{\partial x^2}+G_{\mathrm{VH}}\frac{\partial^2 u}{\partial z^2}+(D_{13}+G_{\mathrm{VH}})\frac{\partial^2 w}{\partial x\,\partial z}$$
$$\rho\frac{\partial^2 w}{\partial t^2}=G_{\mathrm{VH}}\frac{\partial^2 w}{\partial x^2}+D_{33}\frac{\partial^2 w}{\partial z^2}+(D_{13}+G_{\mathrm{VH}})\frac{\partial^2 u}{\partial x\,\partial z} \tag{9}$$

For computational convenience define

$$\alpha^2=\frac{D_{33}}{\rho},\quad \beta^2=\frac{G_{\mathrm{VH}}}{\rho},\quad \eta=\frac{\beta}{\alpha}$$
$$\lambda^2=\frac{D_{11}}{D_{33}},\quad \mu=\frac{D_{13}}{D_{33}},\quad b=\frac{1-\eta^2}{\mu+\eta^2} \tag{10}$$

It can then readily be verified that, due to the constraint relationship (equation (6))

$$\lambda^2-(\mu+\eta^2)/b=\eta^2 \tag{11}$$

To uncouple equations (9) two potential functions $N(x,z)$ and $H(x,z)$ are introduced, related to u and w as follows

$$u=\frac{\partial N}{\partial x}+b\frac{\partial H}{\partial z}$$
$$w=b\frac{\partial N}{\partial z}-\frac{\partial H}{\partial x} \tag{12}$$

Substituting (12) in (9) while accounting for (10) and (11) leads, after some simple operations, to

$$\lambda^2\frac{\partial^2 N}{\partial x^2}+\frac{\partial^2 N}{\partial z^2}=a^{-2}\frac{\partial^2 N}{\partial t^2}$$
$$\frac{\partial^2 H}{\partial x^2}+\frac{\partial^2 H}{\partial z^2}=\beta^{-2}\frac{\partial^2 H}{\partial t^2} \tag{13}$$

The general harmonic solution of (13) is

$$N=(A'\,\mathrm{e}^{ihmz}+A''\,\mathrm{e}^{-ihmz})\,\mathrm{e}^{i(\omega t-hlx)}$$
$$H=(B'\,\mathrm{e}^{ikqz}+B''\,\mathrm{e}^{-ikqz})\,\mathrm{e}^{i(\omega t-kpx)} \tag{14}$$

where $i=\sqrt{-1}$, provided that the parameters l, m, p and q, being in general complex numbers, satisfy

$$\lambda^2 l^2+m^2=1 \tag{15}$$
$$p^2+q^2=1 \tag{16}$$

In the above equations ω is the frequency of vibration (in rad/s), $h=\omega/\alpha$ and $k=\omega/\beta$. It is easy to check by direct substitution that equations (14) (along with equations (15) and (16)) constitute a solution of equation (13). A', A'', B', B'' in (14) are arbitrary constants of integration, to be determined from the boundary conditions of the problem.

Using equations (14), (12), (8), (10) and (4), expressions for u, w, σ_z and τ_{xz} can be derived in terms of the same constants of integration A', A'', B', B'' (shown in matrix form in Table 2).

Having expressed stresses (σ_z,τ_{xz}) and displacements (u,w) in terms of four constants for each layer, a total of $4n$ equations are needed to determine all the unknown quantities, if n is the number of soil layers in the deposit. These equations are provided by the boundary conditions at each layer interface and at the loaded surface. The analysis is identical with the one described by Gazetas (1980a) in connection with a soil deposit consisting of heterogeneous isotropic layers, although the basic solutions for u, w, σ_z and τ_{xz} within each layer are different in the two cases.

Table 2. Expressions for u, w, σ_2 and τ_{xz}; $f(z)=\mathrm{e}^{ihmz}$ and $g(z)=\mathrm{e}^{ikqz}$

$$\begin{Bmatrix}\sigma_z\\ \tau_{xz}\\ u\\ w\end{Bmatrix}=\begin{bmatrix}-h^2(\mu l^2+bm^2)D_{33}f(z) & k^2pq(b\mu-1)D_{33}g(z) & -h^2(\mu l^2+bm^2)D_{33}/f(z) & -k^2pq(b\mu-1)D_{33}/g(z)\\ h^2ml(1+b)G_{\mathrm{VH}}f(z) & k^2(p^2-bq^2)G_{\mathrm{VH}}g(z) & -h^2ml(1+b)G_{\mathrm{VH}}/f(z) & k^2(p^2-bq^2)G_{\mathrm{VH}}/g(z)\\ -ihlf(z) & ikqbg(z) & -ihl/f(z) & -ikqb/g(z)\\ ihmbf(z) & ikpg(z) & -ihmb/f(z) & ikp/g(z)\end{bmatrix}\cdot\begin{Bmatrix}A'\\ B'\\ A''\\ B''\end{Bmatrix}\mathrm{e}^{i(\omega t-hlx)}$$

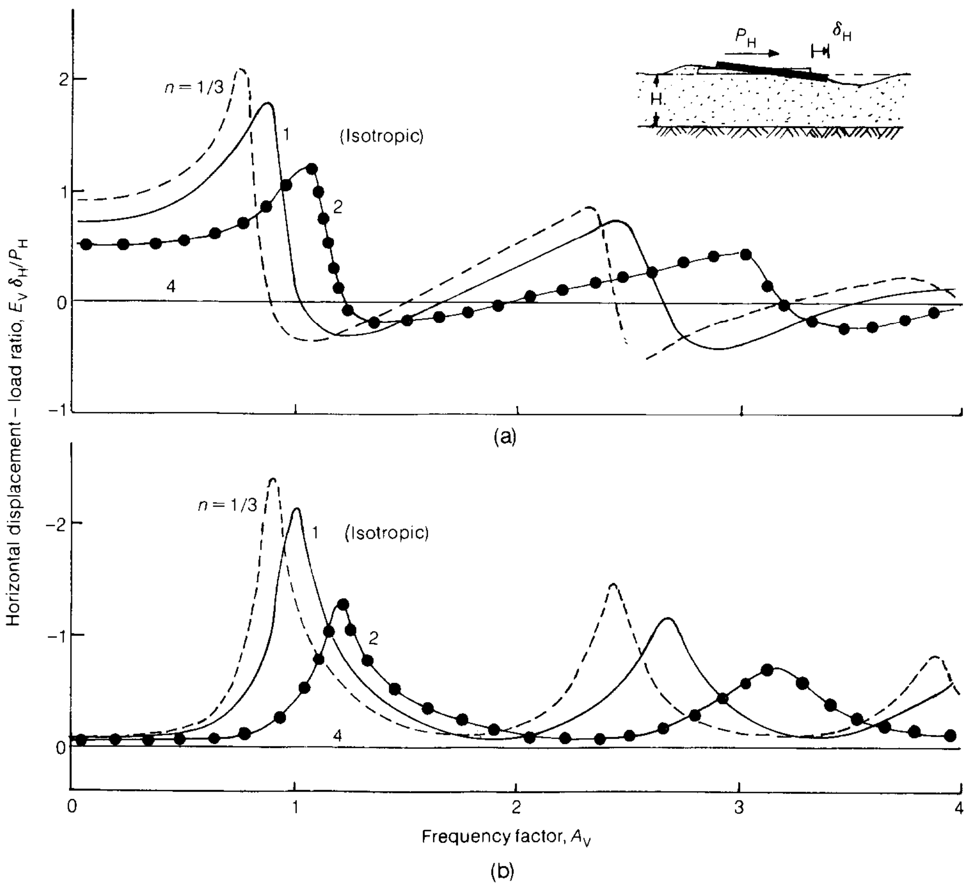


Fig. 2. Undrained dynamic horizontal displacement–load ratio ($H/B = 1$): (a) in-phase component; (b) 90%-out-of-phase component

PRESENTATION OF RESULTS

Results are obtained for vertical, sliding and rocking vibrations of rigid massless footings subjected to vertical, horizontal and moment loading varying harmonically with time. By allowing the frequency of vibration to vanish, general solutions for the static displacements are also readily derived. Only two extreme types of mechanical behaviour of the soil–footing interface, corresponding to adhesive or frictionless contact, are discussed in the Paper since several analyses have confirmed that a contact obeying Coulomb's friction law leads to intermediate response amplitudes. Also, since layering conditions vary from site to site, only two simple, characteristic soil models, namely, a homogeneous cross-anisotropic halfspace and a homogeneous cross-anisotropic stratum underlain by rigid rock, are examined here. These models represent extreme categories of actually encountered soil profiles and their study can offer considerable insight into the dynamics of cross-anisotropic soils.

The results are displayed in the form of normalized displacement–load amplitude ratios (hereafter called compliances) as functions of dimensionless groups of key material and geometric parameters

$$\frac{E_V\,\delta_V}{P_V} \quad \text{or} \quad \frac{E_V\,\delta_H}{P_H} \quad \text{or} \quad \frac{E_V\,B^2\,\phi_o}{M_o} = \mathrm{f}\left(\frac{H}{B}, n, \nu_{VH}, \nu_{HH}\,A_V, \xi\right) \tag{17}$$

in which δ_V, δ_H and ϕ_o are the amplitudes of the vertical, horizontal displacements and the angle of rotation of the footing caused by harmonic forces of amplitudes (per unit length) P_V, P_H and M_o respectively; B is half the foundation width and H the thickness of the soil deposit; E_V, n, ν_{VH} and ν_{HH} are the independent anisotropic soil parameters; A_V is a dimensionless frequency factor defined as

$$A_V = \omega B/\sqrt{(E_V/\rho)} \tag{18}$$

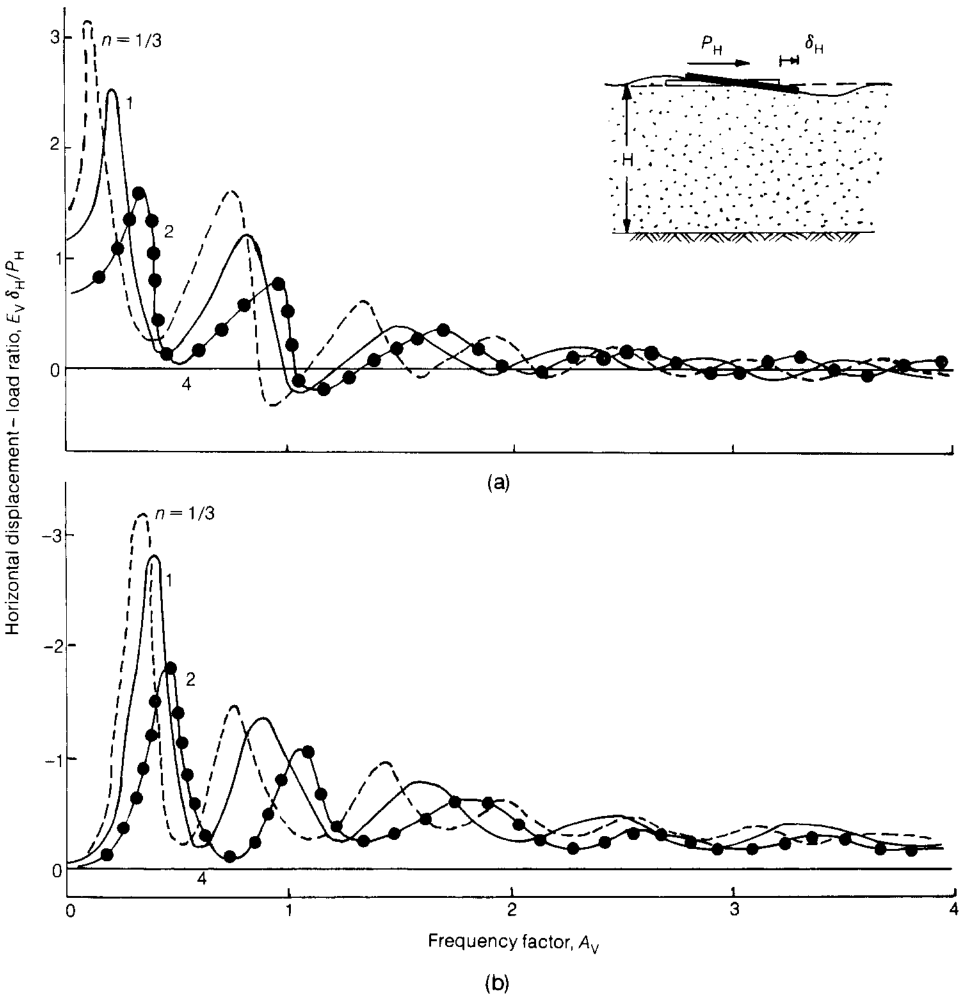


Fig. 3. Undrained dynamic horizontal displacement–load ratio ($H/B = 3$): (a) in-phase component; (b) 90%-out-of-phase component

and ξ is the critical damping ratio which reflects the internal energy dissipation in soil due to hysteresis and friction.

Each displacement consists of two components; one in phase and one 90° out of phase with the applied harmonic load. The first represents the recoverable, elastic component of deformation while the second expresses the dissipation of energy by waves propagating away from the foundation (radiation or geometric damping) and by hysteresis and friction in the soil (internal damping).

PARAMETRIC STUDY: UNDRAINED RESPONSE

Study of the undrained foundation response is of particular geotechnical engineering interest since initial displacements, caused by static loads during or immediately after construction, take place due to undrained shearing deformations of water-saturated clays that may exist in the soil deposit. Moreover, dynamic loads involve short time intervals between imposed stress changes and thus undrained conditions prevail in most saturated soils (see, for example, Richart *et al.*, 1970).

Under undrained conditions saturated soil behaves as an incompressible solid and the two independent Poisson's ratios, ν_{VH} and ν_{HH}, are given by equations (3). Thus, the degree of material anisotropy is uniquely described with the ratio $n = E_H/E_V$, since the shear modulus ratio $m = G_{VH}/E_V$ can be computed in terms of n from the constraint relationship (6).

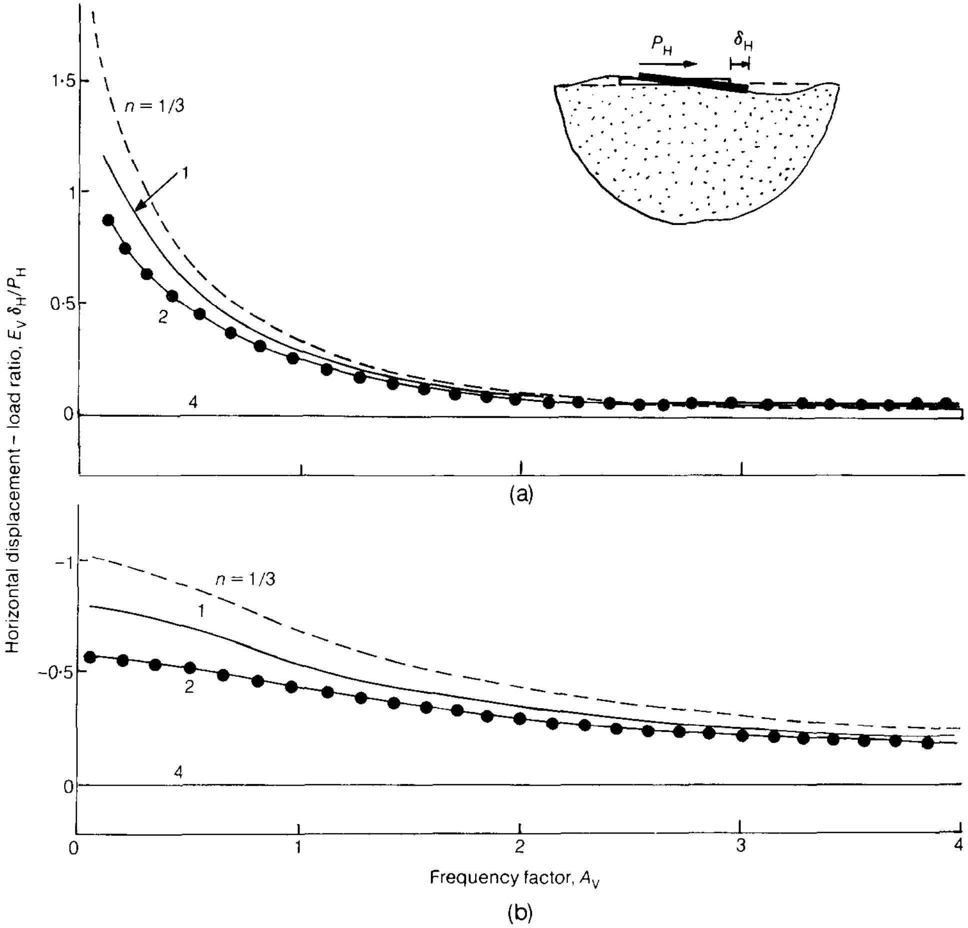


Fig. 4. Undrained dynamic horizontal displacement–load ratio ($H/B = \infty$): (a) in-phase component; (b) 90%-out-of-phase component

Horizontal loading

Figures 2, 3 and 4 portray the horizontal displacement–load ratio $E_V\,\delta_H/P_H$ (horizontal compliance) as a function of the frequency factor A_V and the degree of cross-anisotropy n, for three homogeneous soil deposits. A wide range of possible thicknesses is covered, from $H/B = 1$ (very shallow deposit) to $H/B = \infty$ (very deep deposit). Both in-phase and 90°-out-of-phase displacement components are shown for a critical damping ratio $\xi = 0{\cdot}05$. Adhesive contact is assumed between footing and soil. Several conclusions can be drawn from these figures.

Layer thickness. Regarding the effect of layer thickness, for a given degree of anisotropy, it is evident that the existence of rigid bedrock at relatively shallow depths drastically reduces the static and low-frequency foundation displacements. This is better illustrated in Fig. 5 depicting

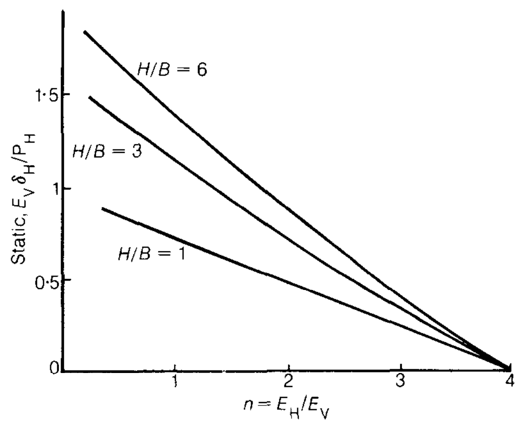


Fig. 5 Undrained static horizontal displacement–load ratio as a function of the degree of ansiotropy n, for various values of the H/B ratio

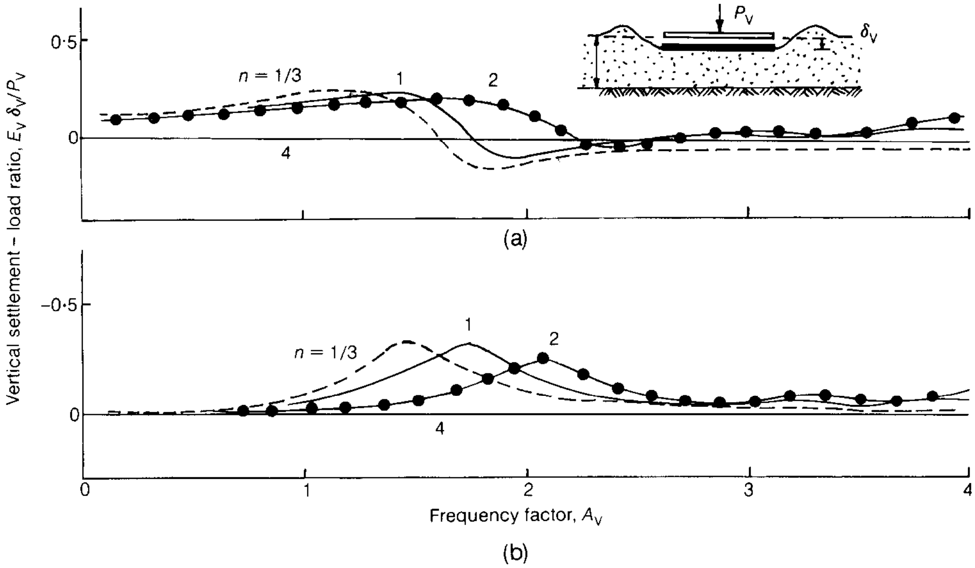


Fig. 6. Undrained dynamic vertical displacement–load ratio ($H/B = 1$): (a) in-phase component; (b) 90%-out-of-phase component

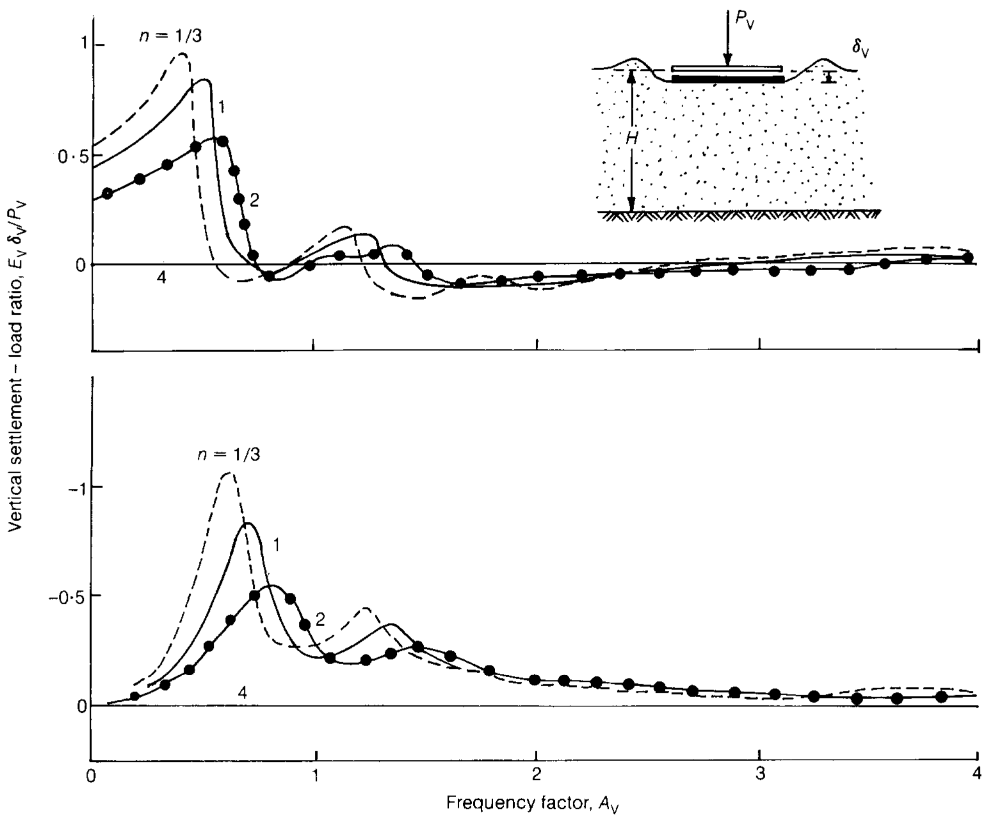


Fig. 7. Undrained dynamic vertical displacement–load ratio ($H/B = 3$): (a) in-phase component; (b) 90%-out-of-phase component

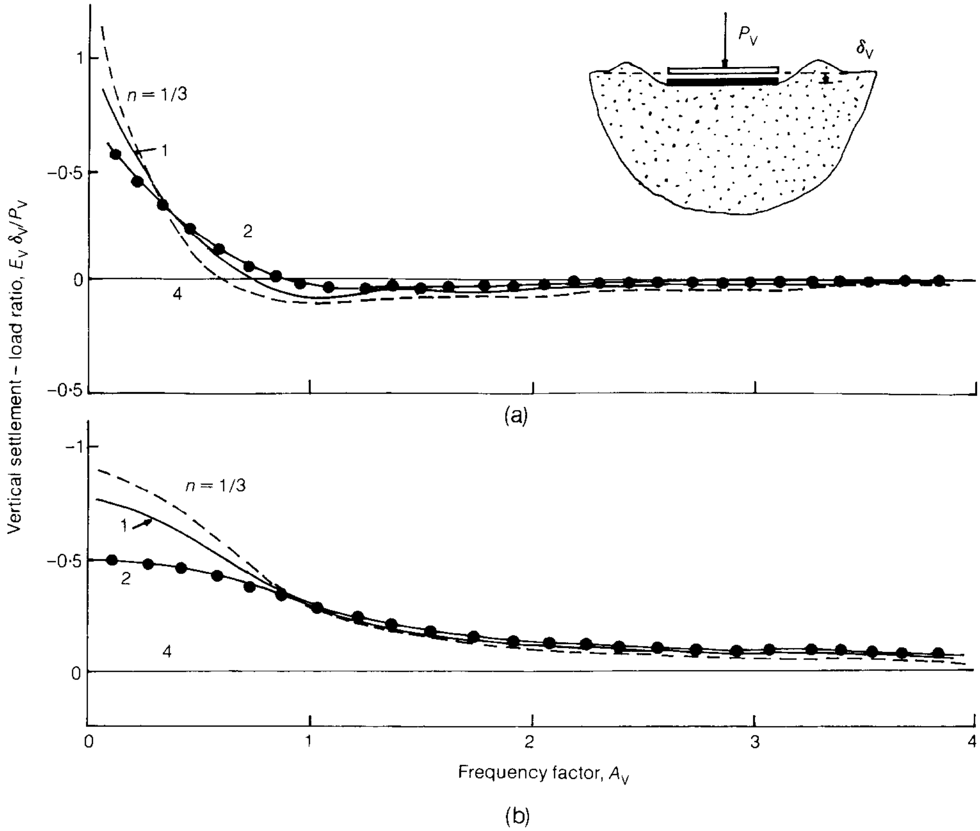


Fig. 8. Undrained dynamic vertical displacement–load ratio ($H/B = \infty$): (a) in-phase component; (b) 90%-out-of-phase component

the horizontal static compliance as a function of the degree of anisotropy n for several values of the H/B ratio. With an infinitely thick deposit (i.e. halfspace) δ_H tends to infinity as A_V tends to zero, in agreement with classic theory of elasticity (e.g. Poulos & Davis, 1974).

The variation of dynamic displacements with frequency reveals an equally strong dependence on H/B. On a stratum, both in-phase and 90°-out-of-phase components of displacement are not smooth and monotonically decreasing functions of frequency, as on a halfspace, but exhibit several peaks which are the product of resonance phenomena: waves propagating away from the foundation reflect at the soil–bedrock interface and return back to the surface. As a result, foundation motion significantly increases at specific frequencies of vibration which, as shown later, are close to the natural frequencies of the soil deposit.

In the low frequency range, below the first resonant frequency, as long as bedrock does exist at some depth below the surface (i.e. $H \neq \infty$), the 90°-out-of-phase component of displacement is negligibly small, especially when compared with the corresponding halfspace displacements. This is due to the fact that no surface waves can be created in a soil stratum at these frequencies; thus no radiation damping is present and the said displacement component reflects only the internal damping in the soil.

Cross-anisotropy. It is evident from Figs 2 to 4 that, relative to a corresponding isotropic deposit, the following effects can be ascribed to cross-anisotropy.

As the ratio n of horizontal to vertical modulus increases, the static horizontal compliance decreases—an anticipated phenomenon that is more clearly illustrated in Fig. 5, for several H/B ratios. In the extreme case of $n = 4$ the medium behaves as irrotational, in addition to being incompressible, and consequently no deformation occurs, as was first pointed out by Gibson (1974).

The importance of anisotropy increases as the deposit becomes thicker (Fig. 5). In other words, the larger H/B is, the faster the static δ_H decreases with increasing n. For H/B between 1 and 6 and n

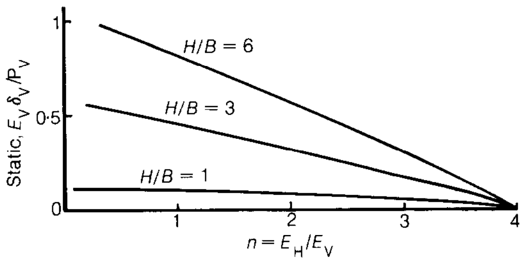


Fig. 9. Undrained static vertical displacement–load ratio as a function of the degree of anisotropy *n*, for various values of the *H*/*B* ratio

between 0·50 and 2, the simple expression

$$\delta_{\mathrm{H\,static}} = \frac{5P_{\mathrm{V}}}{8E_{\mathrm{V}}}\;\frac{4{\cdot}10 - n(H/B)^{0{\cdot}1}}{1+(5/3)(B/H)} \tag{19}$$

for

$$1 \leqslant H/B \leqslant 6$$
$$0{\cdot}5 \leqslant n \leqslant 2{\cdot}5$$

fits the numerical data with very good accuracy (error less than 4%).

The second effect of increasing *n* ratio is to increase the resonant frequencies and decrease the resonant amplitudes for all soil profiles but the halfspace. For each deposit the first resonant frequency can be approximated with reasonable accuracy by the fundamental natural frequency of the deposit in shear vibration. The latter, obtained as for a cantilever shear beam (see, for example, Newmark *et al.*, 1974), is given by

$$\omega_{\mathrm{f}} = \frac{\pi}{2}\frac{\beta}{H} \tag{20}$$

and leads to a resonant frequency factor

$$A_{\mathrm{V,r}} = \frac{\pi}{2}\frac{B}{H}(4-n)^{-1/2} \tag{21}$$

if account is taken of the constraint relation in undrained conditions. As an example, for $H/B = 1$ and $n = \frac{1}{3}$, 1 and 2, equation (21) yields $A_{\mathrm{V,r}} = 0{\cdot}82$, 0·907 and 1·111, respectively, which compare favourably with the values 0·78, 0·86 and 1·08 read from Fig. 3.

Figures 2–5 and equations (19) and (21) can be utilized in practice to predict the performance of a variety of foundations and structures subjected to static and dynamic horizontal loads. The latter may either be applied directly on the structure and then transmitted into the ground through the foundation (as with machine, wind or sea-wave loads) or transmitted from the ground to the structure (as with earthquake and traffic-induced loads). Procedures that employ the compliance functions to determine the response of foundations and structures to such dynamic loads are well established and can be found in numerous publications (Richart *et al.*, 1970; Ratay, 1971; Yoshimi *et al.*, 1977; American Society of Civil Engineers, 1979).

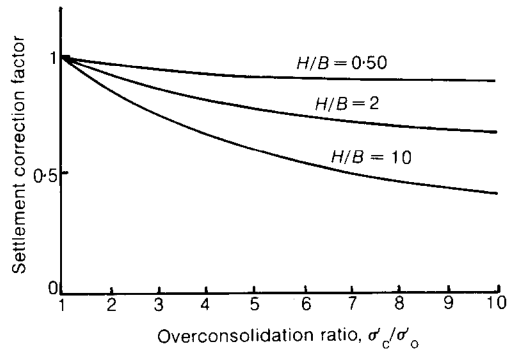


Fig. 10. Settlement correction for three-dimensional effects on initial excess pore pressure distribution (adapted from Highway Research Board, 1973)

Vertical loading

The vertical settlement–load ratio $E_{\mathrm{V}}\,\delta_{\mathrm{V}}/P_{\mathrm{H}}$ is displayed in Figs 6–8 as a function of A_{V} and *n*, for three homogeneous soil deposits having $H/B = 1, 3$ and ∞, respectively. Adhesive contact is again assumed between footing and soil, while $\xi = 0{\cdot}05$.

Several differences in the response of a foundation to vertical and horizontal loads are evident from a comparison of Figs 6–8 with Figs 2–4.

Vulnerability. For a given soil deposit, foundations are more vulnerable to horizontal than vertical statically applied loads, since they experience horizontal displacements larger, by a factor of at least 2, than the settlements due to a vertical load of equal magnitude.

Layer thickness. Static settlements exhibit a stronger dependence on layer thickness but are less sensitive to the degree of soil anisotropy than horizontal displacements are. This is better illustrated in Fig. 9, as compared with Fig. 5. It can be seen that foundations on very shallow layers (e.g. $H/B = 1$) settle by an amount which is essentially independent of *n*. The phenomenon can be attributed to the one-dimensional nature of deformations that take place in shallow deposits under the central part of relatively large loading areas (i.e. in cases of small *H*/*B* ratio). It seems reasonable to argue that such deformations depend primarily on E_{V} and ν_{VH}; E_{H} affects only the deformations under the foundation perimeter and its importance diminishes with *H*/*B*. Thus *n* is unimportant in such cases.

The phenomenon is reminiscent of the develop-

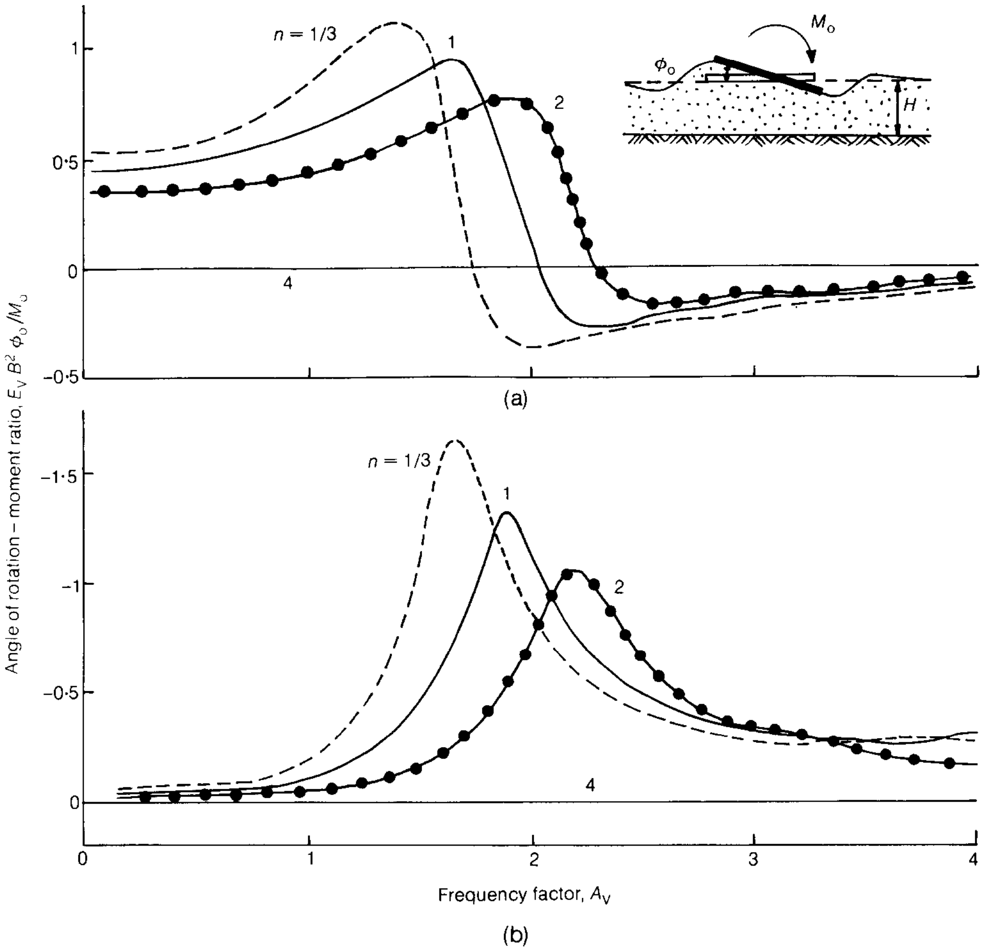


Fig. 11. Undrained dynamic angle of rotation–moment ratio ($H/B = 1$): (a) in-phase component; (b) 90%-out-of-phase component

ment of excess pore pressures in clayey strata subjected to axisymmetric loading, as discussed by Skempton & Bjerrum (1957). Figure 10 shows the dependence on overconsolidation (OCR) and H/B ratios of the correction factor which should multiply the one-dimensionally determined consolidation settlement in order to compensate for three-dimensional (3-D) effects on the initial excess pore-water pressure distribution. It is clear that in shallow strata ($H/B < 1$) the 3-D correction is minor and, moreover, almost independent of OCR. Certainly, under plain-strain loading conditions, as is the case here, 3-D effects would be even smaller and thus settlement would practically be independent of the n ratio which, in general, increases with increasing OCR (e.g. Gibson, 1974).

For relatively shallow deposits ($1 \leqslant H/B < 4$), the simple expression

$$\delta_{V,\text{static}} = \frac{P_V}{45E_V}(4-n)^{(1/6)(H/B)}\left(1+3{\cdot}5\frac{H}{B}\right) \quad (22a)$$

with

$$1 \leqslant H/B < 4$$
$$0{\cdot}5 \leqslant n \leqslant 2{\cdot}5$$

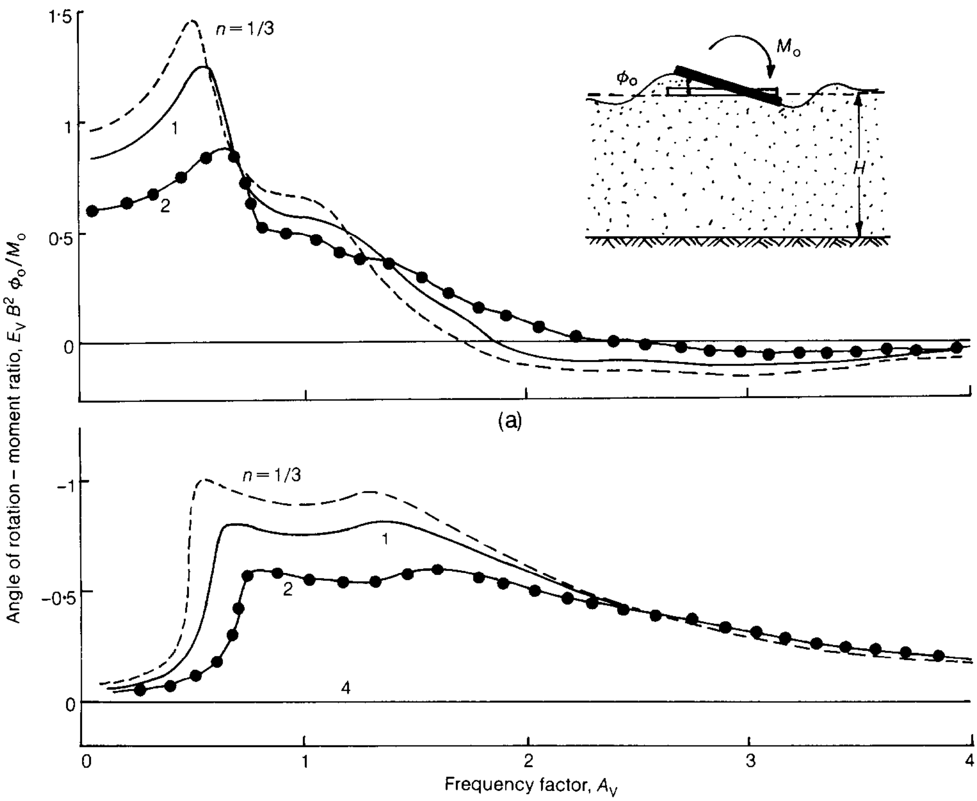
fits the numerical data with reasonable accuracy (error within 10%); for deeper deposits the approximation takes the form

$$\delta_{V,\text{static}} = \frac{2P_V}{5E_V}(4-n)\log_{10}\frac{H}{B} \quad (22b)$$

with

$$H/B \geqslant 8$$

all n values



(b)

Fig. 12. Undrained dynamic angle of rotation–moment ratio ($H/B = 3$): (a) in-phase component; (b) 90%-out-of-phase component

Contrast equation (22b) with the expression for the undrained settlement of a uniformly loaded circular area given by Hooper (1975) (on the basis of results by Gerrard & Harrison, 1970a). Use of the constraint relationship reduces that expression to

$$\delta_{V,\text{static}} = \frac{P_V}{2\pi R E_V}(4-n) \qquad (23)^2$$

with

$$H/B = \infty$$

all n values

which reveals a similarly strong dependence on settlement on n for both circular and strip footings on very deep homogeneous soil deposits. Furthermore, introducing the constraint relationship in equation (2) (derived by Gibson & Sills, 1975), yields for a linearly heterogeneous cross-anisotropic halfspace

$$\delta_{V,\text{static}} = \frac{P_{V,\text{av}}}{2r}(4-n) \qquad (24)$$

in which $P_{V,\text{av}}$ is the average vertical pressure on the footing and r the rate of soil heterogeneity (equation (1)). A similar dependence of undrained settlement on the degree of anisotropy n for both homogeneous and heterogeneous deep soil deposits can be inferred from equation (24).

Resonance. Resonance phenomena are again observed at one or two frequencies of vibration but the corresponding peaks are not as sharp as those of the horizontal displacements. In fact, on very shallow deposits ($H/B = 1$) a single flat resonance takes place, which is characteristic of a highly damped system.

A possible explanation of such behaviour stems

[2] P_V in equation (23) is total applied force on a circular area, whereas P_V in the previous equations is total force per unit length on a strip footing. R = the radius of the foundation.

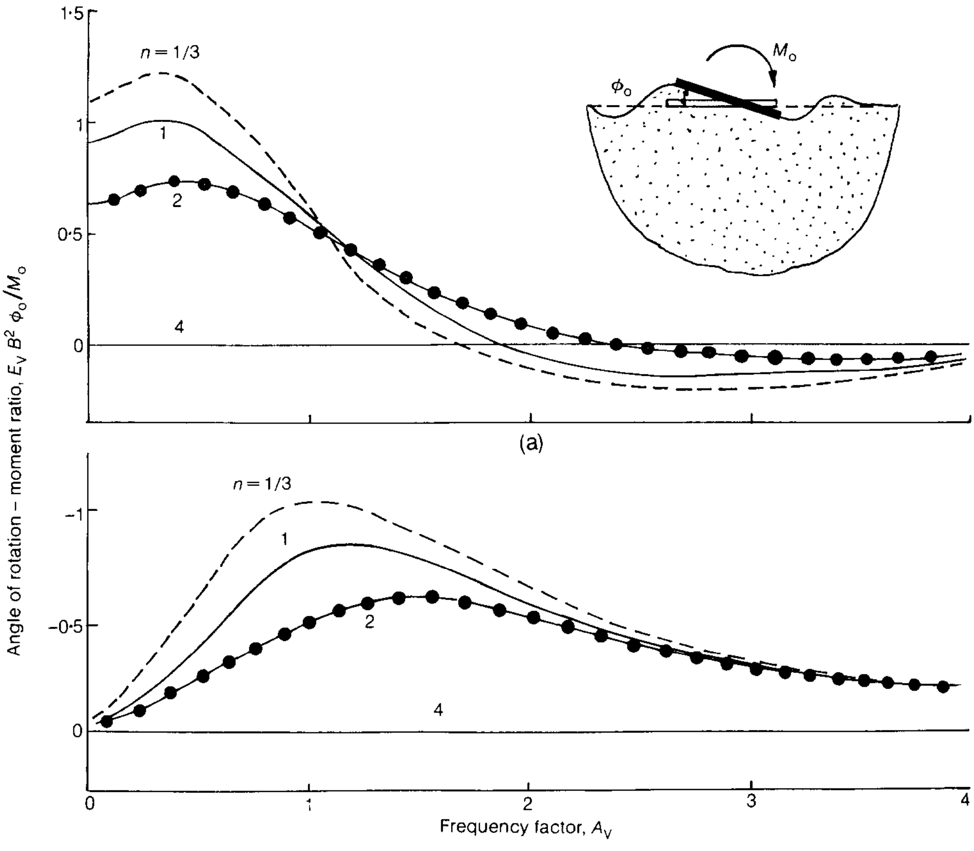


Fig. 13. Undrained dynamic angle of rotation–moment ratio ($H/B = \infty$): (a) in-phase component; (b) 90%-out-of-phase component

from the fact that both compressional and shear waves participate in the motion at first resonance which occurs at a frequency factor

$$A_{V,r} \simeq \pi \frac{B}{H}(4-n)^{-1/2} \quad (25)$$

that lies in between the fundamental natural frequency factors of the deposit in pure shear (equation (21)) and in pure dilatation ($A_{V,r} = \infty$, due to incompressibility of the material). Surface waves are also present during resonance, as a result of the interference of the two types of waves; the ensuing radiation damping contributes to further limiting the peak amplitudes of motion (Gazetas & Roesset, 1979).

Moment loading

The response of a rigid foundation to a harmonic moment $M_0 e^{i\omega t}$ is described in Figs 11–14 through the normalized dynamic angle-of-rotation–moment ratio $E_V B^2 \phi_0/M_0$.

The static value of the ratio converges to a finite value (e.g. 0·92, for $n = 1$) as the depth of the deposit grows beyond any limit, in contrast with the static horizontal and vertical displacement–load ratios that tend to infinity as the stratum tends to become a halfspace (Figs 4, 8). Increasing layer depth beyond a value corresponding to $H/B = 3$ has practically no effect on rotation. This implies that the stress and strain fields caused by moment loads are of limited extent, thus influencing only the near-surface soil (Gazetas & Roesset, 1976; Gazetas, 1980a).

One relatively flat resonance takes place at a frequency factor $A_{V,r}$ which can be approximated by equation (25). This indicates that an interference between dilatational and shear waves is responsible for the observed peak, much in a similar way as with vertical vibrations. However, the effect of soil anisotropy is evident in rocking vibrations even at

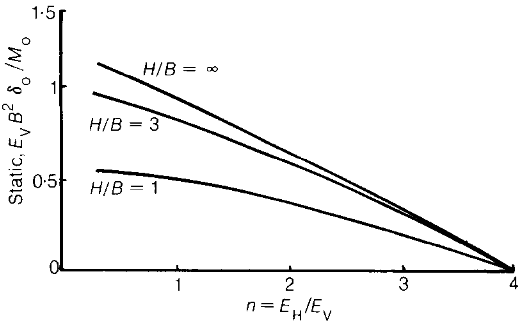


Fig. 14. Undrained static angle of rotation–moment ratio as a function of the degree of anisotropy n, for various values of the H/B ratio

very shallow deposits (e.g. $H/B = 1$) and small frequencies of oscillation (e.g. $A_V < 1·5$); this is hardly the case with vertical motion, as discussed previously (Fig. 6).

EFFECT OF FRICTIONLESS CONTACT

Under static loading conditions no secondary stresses develop in the soil–foundation interface if the material is incompressible. That is, no shear tractions are generated during vertical and moment loading and no normal tractions during horizontal loading. Under dynamic excitation these conclusions appear to be true for horizontal loading throughout the frequency range examined; the response curves corresponding to the two types of contact behaviour (i.e. allowing or not allowing

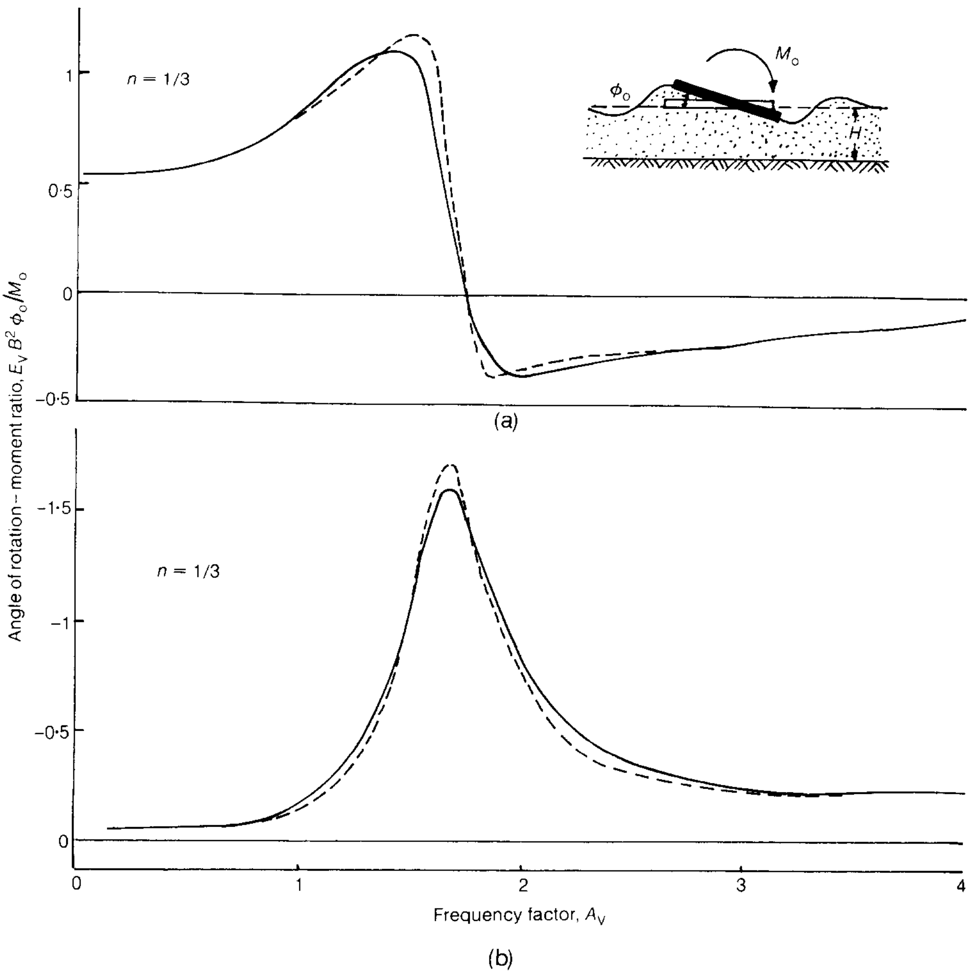


Fig. 15. Effect of frictionless contact on dynamic angle of rotation–moment ratio ($H/B = 1, n = 1/3$); solid line = adhesive contact, dotted line = frictionless contact; (a) in-phase component; (b) 90%-out-of-phase component

secondary tractions to develop) are therefore essentially identical for all the profiles examined.

However, dynamic shear tractions do arise during resonance in vertical and rocking vibrations of footings adhesively connected to the soil surface. Such tractions are not present under an ideally smooth (frictionless) footing. As a consequence, the peaks of the frictionless response curves occur at slightly higher frequencies, i.e. they move away from the fundamental natural frequency of the deposit in shear vibrations. Figure 15 illustrates this observation as it applies to rocking vibrations (Vertical vibration curves for frictionless and adhesive contact show similar differences.) Figure 15 also shows the somewhat higher peaks of the frictionless curves. Overall the observed discrepancies are rather insignificant and, furthermore, decrease with increasing degree of anisotropy n.

SUMMARY AND CONCLUSIONS

The Paper has presented a rigorous solution to the problem of determining the static and dynamic response of rigid strip footings resting on the surface of a soil deposit consisting of any number of cross-anisotropic soil layers. Vertical, horizontal and moment loading has been considered and the results of comprehensive parametric studies have been presented in the form of normalized displacement–load ratios as functions of dimensionless geometric and material parameters. These results can be readily used in practice to study the performance of foundations and structures in a variety of static and dynamic loading environments. Simple characteristic soil profiles and undrained soil behaviour have been considered, although the method can treat as easily any horizontally layered deposit and drained conditions.

Three main factors influence the normalized displacement–load ratios of rigid foundations on a cross-anisotropic soil stratum

(a) the stratum depth to foundation halfwidth ratio H/B
(b) the degree of soil anisotropy, measured with the ratio $n = E_H/E_V$
(c) the dimensionless frequency factor A_V

The importance of these factors has been demonstrated through a number of parametric plots for all three types of loading and through extensive comparisons with pertinent results of other researchers for isotropic and anisotropic soils. On the basis of the presented numerical data, simple and sufficiently accurate formulas have been developed and offered in the Paper for static displacements and resonant frequencies in terms of n and H/B.

The results of these studies suggest that soil anisotropy greatly influences both static and dynamic foundation displacements under undrained conditions. Soils characterized by large n values are likely to experience static settlements as much as 50% smaller than what computations based on classical isotropic theories indicate; this may well be the reason for the usual overprediction of settlements on heavily overconsolidated clays. On the other hand, soils exhibiting n values smaller than unity, like sands and sensitive clays, settle more than isotropic soils with the same vertical stiffness.

Finally, when the dynamic performance of foundations is studied, neglecting soil anisotropy may lead to unsafe conclusions regarding the possibility of resonance and the response at high frequencies of oscillation.

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APPENDIX 1

DRAINED G_{VH}/E_V OF LONDON CLAY

Drained laboratory triaxial tests carried out by Atkinson (1975) indicated that $n' = 2$, $\nu_{VH}' = 0{\cdot}19$ and $\nu_{HH}' = 0{\cdot}00$, where the primes denote drained parameters. For a saturated poro-elastic soil, Uriel & Kañizo (1971) give the following relationship for undrained n

$$n = \frac{2n'(1-\nu_{HH}'-2n'\nu_{VH}'^2)}{1-2n'\nu_{VH}'+n'-n'^2\nu_{VH}'^2-2n'\nu_{VH}'\nu_{HH}'-\nu_{HH}'^2} \tag{26}$$

which, upon substitution of the drained values, yields $n = 1{\cdot}633$. Assuming similar deformational behaviour of the soils at Barbican Arts Centre and at Ashford, the undrained value of G_{VH}/E_V can be estimated as equal to 0·372 (Table 1). Furthermore, the condition that the shear moduli G_{VH} and G_{HH} remain constant during consolidation leads to a ratio of drained and undrained horizontal moduli (see Hooper, 1975).

$$\frac{E_H}{E_H'} = \frac{4-n}{2(1+\nu_{HH}')} \simeq 1{\cdot}18$$

The vertical moduli ratio is then

$$\frac{E_V}{E_V'} = \frac{E_H}{E_H'}\frac{n'}{n} \simeq 1{\cdot}18 \times \frac{2}{1{\cdot}633} \simeq 1{\cdot}445$$

and consequently

$$\frac{G_{VH}'}{E_V'} = \frac{G_{VH}}{E_V}\frac{E_V}{E_V'} = 0{\cdot}372 \times 1{\cdot}445 \simeq 0{\cdot}538$$

which is the value shown in Table 1.